

**Measuring Densities of Solids and Liquids Using Magnetic Levitation:
Fundamentals**

SUPPORTING INFORMATION

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General Methods

The NdFeB magnets (5 cm × 5 cm × 2.5 cm) were purchased from K&J Magnetics (www.kjmagnetics.com) and aligned on top of one another 4.5 cm apart within aluminum blocks. Similar magnets can also be obtained from Applied Magnets, www.magnet4less.com at a lower price. The strength of the magnetic field within the device was measured using a hand-held DC magnetometer (AlphaLab Inc, www.trifield.com). Calibrated density standards ($\pm 0.0002 \text{ g/cm}^3$ at 23°C) were purchased from American Density Materials (Stauton, VA; www.densitymaterials.com). Spherical polymer samples were purchased from McMaster-Carr (www.mcmaster.com). Polystyrene microspheres with precisely defined radii were supplied by Duke Scientific Corporation (www.dukescientific.com), Polysciences, Inc. (www.polysciences.com), and Spherotech (www.spherotech.com). All other samples and reagents were purchased from Sigma Aldrich (Atlanta, GA) and used without further purification. “Levitation height” of samples was measured using a ruler with millimeter-scaled marking. Helium pycnometry measurements were performed by Quantachrome Instruments for a fee on an Ultrapyc 1200e instrument.

Figure S0. A plot generated using numerical simulation using COMSOL Multiphysics showing the dependence of the z -component of the magnetic field B_z on the separation between magnets (d) for $d = 25, 35, 45, 55, 65, 75$ mm along the centerline between the two magnets.

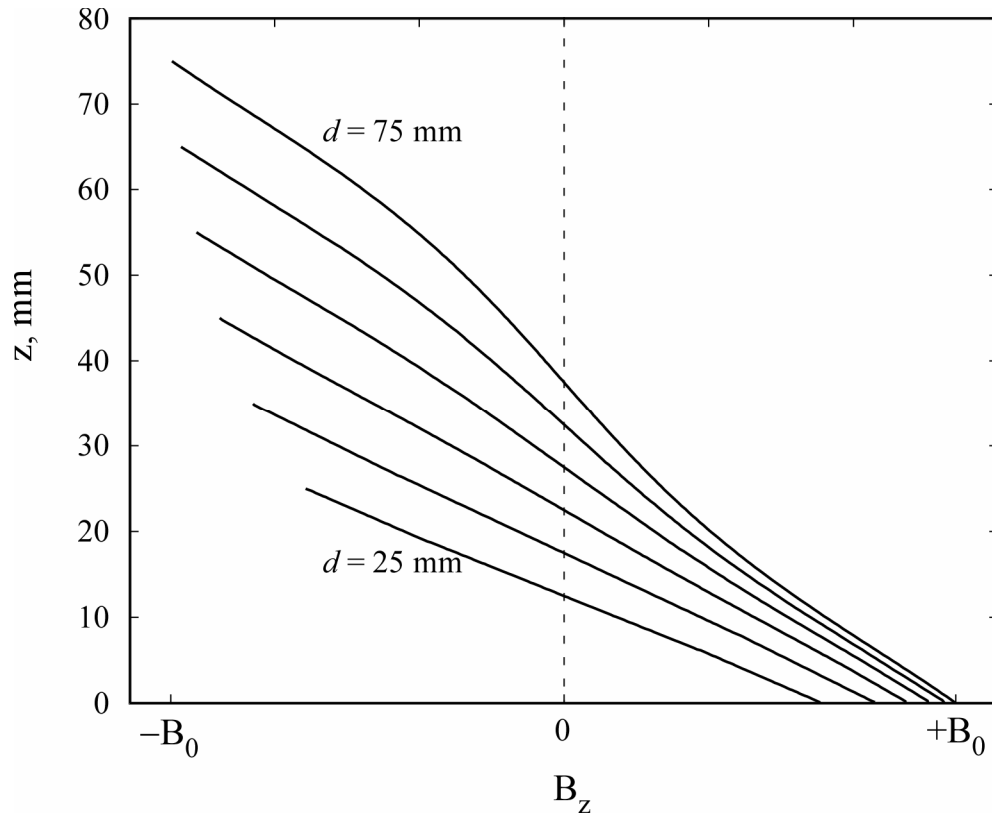


Figure S1. Photographs demonstrating levitation of a glass bead (density = 1.1500 ± 0.0002 g/cm³) in different aqueous paramagnetic solutions (1M MnCl₂, 1M MnBr₂, 1M CuSO₄, 1M GdCl₃, 1M FeCl₂).

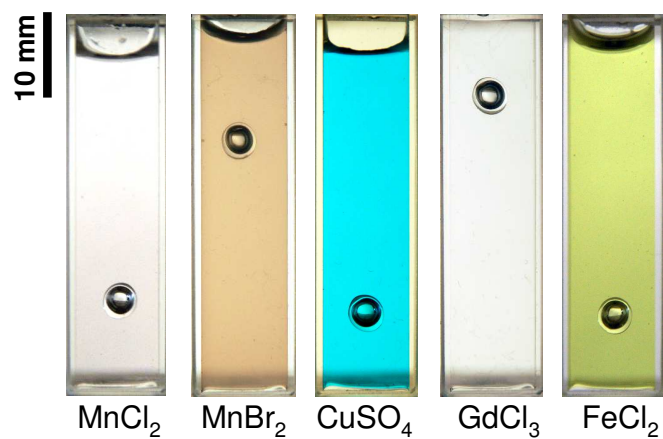


Figure S2. Net Volumetric Magnetic Susceptibilities of Common Diamagnetic Substances¹⁻⁷

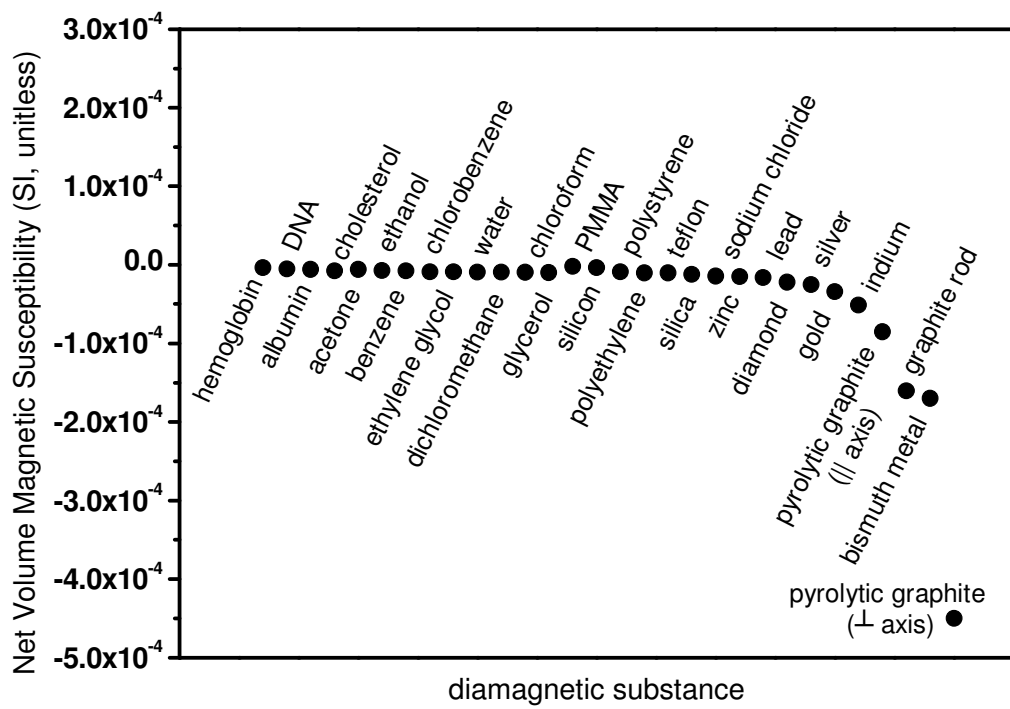


Figure S3. Deliberate misalignment of the container with the centerline between the magnets (red dotted line). Beads of different densities (from top to bottom: 1.0500, 1.0800, 1.1000, 1.1200, 1.1500 g/cm³) levitating in 1M MnCl₂ align with the centerline between the magnets regardless of the position of the container, as long as the centerline is accessible within the container (A and B). Inability of the beads to align with the centerline (C and D) does not result in significant change in the levitation height of the beads. Scale bar represents 10 mm.

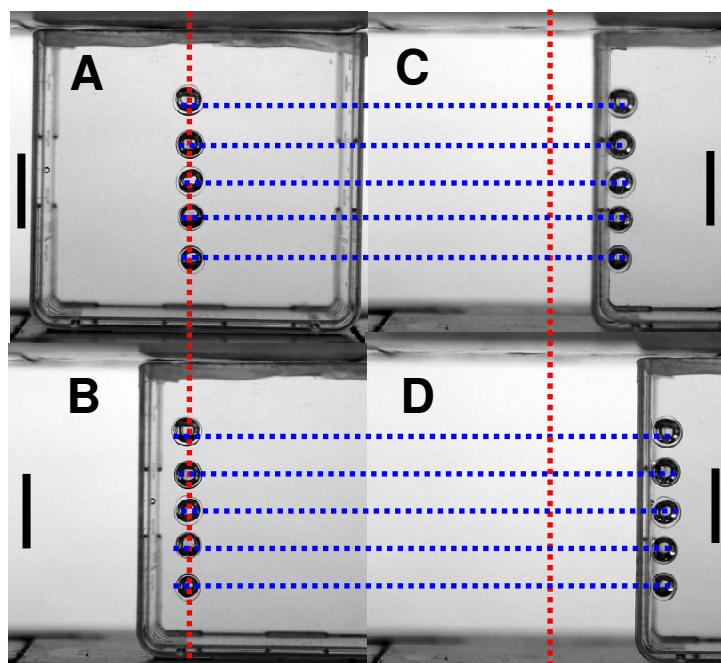


Figure S4. Effect of tilting the experimental set-up on height at which the objects levitate. A) Photographs of beads of different density (from top to bottom: 1.0500, 1.0800, 1.1000, 1.1200, 1.1500 g/cm³) levitating in 1 M MnCl₂ at different values of tilt angle θ . The dimensions of the container in which the beads levitate are 50 mm \times 30 mm \times 45 mm. The container spans the entire width and height of the magnets and is centered lengthwise between the magnets. B) Images in shown in panel A rotated by angle θ to emphasize the effect of tilting on the levitation height of the beads.

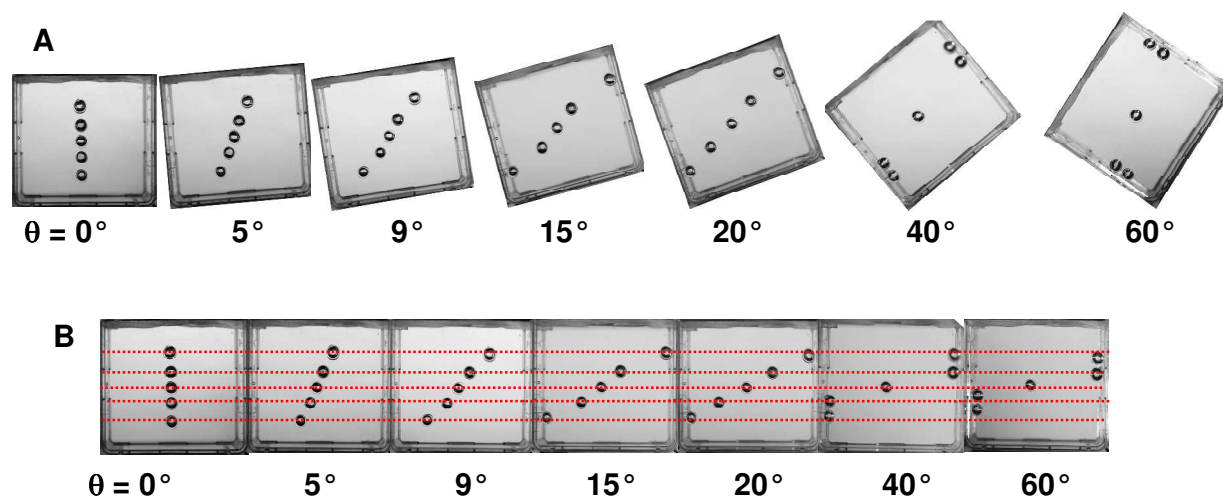
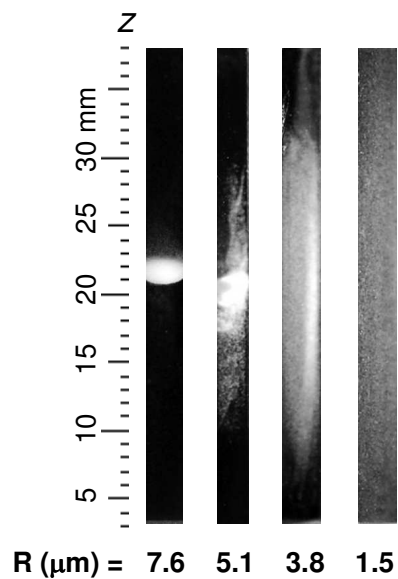


Figure S5. Photographs of polystyrene spheres of different radii levitating at t_0 in aqueous 500 mM MnCl_2 . Spheres of $R > \sim 7 \mu\text{m}$ form well defined clusters at h , spheres of $\sim 2 \mu\text{m} < R < \sim 7 \mu\text{m}$ form diffuse clouds centered around h , while spheres of $R < \sim 2 \mu\text{m}$ remain essentially uniformly dispersed throughout the solution.



Sources and Magnitude of Errors for Calculating Densities of Objects Using Eqn (8)

Experimental parameters and constants: B_0 , d , g , c , and T

We measured the magnitude of the magnetic field at the center on the surface of the bottom magnet with a magnetometer and found that in the configuration shown in **Fig. 1a** B_0 was (0.375 ± 0.003) Tesla.

The distance between the two magnets, d , remained constant in our experiments and was equal to (45 ± 0.5) mm.

The acceleration due to gravity, g , at the surface of the Earth ranges from $9.78 - 9.82$ m/s^2 , where the exact value depends on the latitude and other factors; we used the average value of 9.80 m/s^2 in our calculations and neglected this variation in our error analysis. The typical variation in g does not constitute a significant source of uncertainty in calculation of density using eqn 8.

We measured T using a thermometer with an accuracy of ± 1 °C.

We did not measure c directly—instead, we calculated it (eqn. S0A) based on the mass of the salt m (g) measured using an analytical balance with accuracy of $\delta m = 0.1 \times 10^{-3}$ g (as specified by the manufacturer of the analytical balance), the total volume of solution V (L) measured using a volumetric flask with accuracy of $\delta V = 0.08 \times 10^{-3}$ L (as specified by the manufacturer of the volumetric glassware), and the molecular weight of the salt M (g/mol) as provided by the vendor. We estimated the error associated with our calculation of concentration, δc , using eqn (S0B),⁸ and found it to be less than ± 0.002 M for the range of concentrations we used in this study.

$$c = \frac{m}{MV} \quad (\text{S0A})$$

$$\delta c = \sqrt{\left(\frac{1}{MV} \delta m\right)^2 + \left(-\frac{m}{MV^2} \delta V\right)^2} \quad (\text{S0B})$$

Magnetic Susceptibility of the Medium, χ_m

The magnetic susceptibility of the suspending medium, χ_m , depends on concentrations and magnetic properties of all the species present in solution. We calculated χ_m using eqn (S1), in which χ_p is the molar magnetic susceptibility of the paramagnetic salt (m^3/mol , units of SI), c is the concentration of the paramagnetic salt (mol/L), and -9×10^{-6} is the bulk magnetic susceptibility of water.^{9,10} The dependence of the molar magnetic susceptibilities of the paramagnetic salt χ_p on temperature follows the Curie-Weiss law given by eqn (S2), where T is the temperature of the medium (expressed in $^\circ\text{K}$), C_{CW} is the Curie constant ($\text{m}^3 \text{K} / \text{mol}$) and θ is the Weiss constant ($^\circ\text{K}$) of the paramagnetic salt.¹⁰ The dependence of magnetic susceptibility of the medium on temperature is then given by eqn (S3), in which we assumed that the magnetic properties of water do not change significantly with temperature.^{1,2}

$$\chi_m = \chi_p c - 9 \times 10^{-6} \quad (\text{S1})$$

$$\chi_p = \frac{C_{CW}}{T - \theta} \quad (\text{S2})$$

$$\chi_m = \frac{C_{CW}}{T - \theta} c - 9 \times 10^{-6} \quad (\text{S3})$$

In eqns (S1, S3), we neglected the contribution of the magnetic properties of gasses dissolved in the suspending medium. For example, air-saturated water at room temperature contains up to 0.3 mM of dissolved O_2 gas ($\chi_{\text{O}_2}^{\text{molar}} = 3449 \times 10^{-6}$)^{1,11} — the contribution of O_2 to

the magnetic susceptibility of the medium is $\chi_{O_2} = 1.3 \times 10^{-8}$, which is at least three orders of magnitude less than typical values of χ_m .

Density of the Paramagnetic Solution, ρ_m

The dependence of density of the medium ρ_m on the concentration of paramagnetic salt in solution and on temperature of the solution is given by eqn. (S4), where T is expressed in °C, $\rho_w(T)$ is the density of pure water ($W_0 = 999.65$, $W_1 = 2.0438 \times 10^{-1}$, $W_2 = -6.1744 \times 10^{-2}$), and A – F are empirical, dimensional parameters specific to the paramagnetic salt (for GdCl_3 : $A = 2.538 \times 10^2$, $B = -1.149 \times 10^{-1}$, $C = 1.386 \times 10^{-3}$, $D = -1.306 \times 10^{-1}$, $E = 0$, and $F = 0$; for MnCl_2 : $A = 1.022 \times 10^2$, $B = 4.966 \times 10^{-1}$, $C = -1.307 \times 10^{-2}$, $D = -3.659 \times 10^{-0}$, $E = -1.631 \times 10^{-1}$, and $F = 4.774 \times 10^{-3}$)¹².

$$\begin{aligned} \rho_m(T, c) &= \rho_w(T) + Ac + BcT + CcT^2 + Dc^{3/2} + Ec^{3/2}T + Fc^{3/2}T^2 = \\ &= W_0 + W_1T + W_2T^{3/2} + Ac + BcT + CcT^2 + Dc^{3/2} + Ec^{3/2}T + Fc^{3/2}T^2 \end{aligned} \quad (\text{S4})$$

Magnetic Susceptibility of the Sample, χ_s

The typical values of χ_s for diamagnetic substances are temperature-independent, small (about -1×10^{-6} ... -10×10^{-6}) and vary little from substance to substance (see **Fig. S2** for typical values of magnetic susceptibilities of common substances).^{1, 2, 9} We neglected these differences in calculations of densities based on the empirical estimates of α and β (eqn 8) from the calibration curves. For the measurements of density via the direct application of eqn (8), we set $\chi_s = -5 \times 10^{-6}$ (in the middle of the typical range of values of χ_s for most diamagnetic substances, **Fig. S2**) and estimated the accuracy of this assumption as $\delta\chi_s = 10^{-5}$ (unitless).

This type of oversimplification and the estimation of uncertainty may not be general for all cases, especially when strongly diamagnetic ($|\chi_s| \gg 10^{-5}$) or slightly paramagnetic samples are levitated. In such cases, either an accurate value of χ_s should be used when calculating densities of eqn (8) or a larger margin of error assumed for this type of density measurement.¹³

Levitation Height, h

We measured the levitation heights of objects h using a ruler—we estimate the precision of this measurement to be ± 0.5 mm.

Error Analysis

To estimate the error of our measurements of density using the calibration curves, we assume that we know parameters α and β in eqn (8) exactly, and consider ρ_s to be a function of only one variable h ; we use eqn. (S5) to propagate the uncertainty in h for these types of measurements.⁸ In eqn (S5), α is the calibration parameter from eqn (8) and δh is ± 0.5 mm.

$$\delta\rho_s = \left| \frac{d\rho_s}{dh} \right| \delta h = \alpha \delta h \quad (\text{S5})$$

To estimate the error associated with the direct use of eqn (8) for measuring the density, we treat ρ_s as a function of several independent variables: B_0 , d , χ_s , T , c , and h , each of which is a source of independent random error (eqn. S6).⁸ (Notice that ρ_m and χ_m are not independent parameters – they are calculated from T and c using eqns (S4) and (S3), respectively.)

$$\delta\rho_s = \sqrt{\left(\frac{\partial\rho_s}{\partial T} \delta T \right)^2 + \left(\frac{\partial\rho_s}{\partial c} \delta c \right)^2 + \left(\frac{\partial\rho_s}{\partial \chi_s} \delta \chi_s \right)^2 + \left(\frac{\partial\rho_s}{\partial h} \delta h \right)^2 + \left(\frac{\partial\rho_s}{\partial d} \delta d \right)^2 + \left(\frac{\partial\rho_s}{\partial B_0} \delta B_0 \right)^2} \quad (\text{S6})$$

Eqn. (S7) summarizes eqn. (8) and eqns (S3) and (S4) in a form convenient for calculating the partial derivatives in eqn. (S6).

$$\rho_s = \left(\frac{4(\chi_s - \chi_m)B_0^2}{g\mu_o d^2} \right) h + \left(\rho_m - \frac{2(\chi_s - \chi_m)B_0^2}{g\mu_o d} \right), \quad (\text{S7})$$

where:

$$\rho_m(T, c) = W_0 + W_1 T + W_2 T^{3/2} + A c + B c T + C c T^2 + D c^{3/2} + E c^{3/2} T + F c^{3/2} T^2$$

$$\chi_m = \frac{C_{CW}}{T - \theta} c - 9 \times 10^{-6}$$

The partial derivatives of ρ_m and χ_m with respect to T and c are needed for estimating eqn. (S6) and are given below (eqns S8A-B).

$$\frac{\partial \chi_m}{\partial T} = -C_{CW}c(T-\theta)^{-2} \quad (S8A)$$

$$\frac{\partial \rho_m}{\partial T} = W_1 + \frac{3}{2}W_2T^{1/2} + Bc + 2CcT + Ec^{3/2} + 2Fc^{3/2}T$$

$$\frac{\partial \chi_m}{\partial c} = \frac{C_{CW}}{T-\theta} \quad (S8B)$$

$$\frac{\partial \rho_m}{\partial c} = A + BT + CT^2 + \frac{3}{2}Dc^{1/2} + \frac{3}{2}ETc^{1/2} + \frac{3}{2}FT^2c^{1/2}$$

The partial derivatives of ρ_s with respect to B_0 , d , χ_s , T , c , and h are given by eqns (S9A-F) below.

$$\frac{\partial \rho_s}{\partial T} = -\frac{4B_0^2}{g\mu_o d^2} \left(h - \frac{d}{2} \right) \frac{\partial \chi_m}{\partial T} + \frac{\partial \rho_m}{\partial T},$$

where:

$$\frac{\partial \chi_m}{\partial T} = -C_{CW}c(T-\theta)^{-2} \quad (S9A)$$

$$\frac{\partial \rho_m}{\partial T} = W_1 + \frac{3}{2}W_2T^{1/2} + Bc + 2CcT + Ec^{3/2} + 2Fc^{3/2}T$$

$$\frac{\partial \rho_s}{\partial c} = -\frac{4B_0^2}{g\mu_o d^2} \left(h - \frac{d}{2} \right) \frac{\partial \chi_m}{\partial c} + \frac{\partial \rho_m}{\partial c},$$

where:

$$\frac{\partial \chi_m}{\partial c} = \frac{C_{CW}}{T-\theta} \quad (S9B)$$

$$\frac{\partial \rho_m}{\partial c} = A + BT + CT^2 + \frac{3}{2}Dc^{1/2} + \frac{3}{2}ETc^{1/2} + \frac{3}{2}FT^2c^{1/2}$$

$$\frac{\partial \rho_s}{\partial \chi_s} = \frac{4B_0^2}{g\mu_o d^2} \left(h - \frac{d}{2} \right) \quad (S9C)$$

$$\frac{\partial \rho_s}{\partial h} = \frac{4(\chi_s - \chi_m)B_0^2}{g\mu_o d^2} \quad (S9D)$$

$$\frac{\partial \rho_s}{\partial d} = \frac{2(\chi_s - \chi_m)B_0^2}{g\mu_o} (d^{-2} - 4hd^{-3}) \quad (S9E)$$

$$\frac{\partial \rho_s}{\partial B_0} = \frac{8(\chi_s - \chi_m)}{g \mu_o d^2} \left(h - \frac{d}{2} \right) B_0 \quad (\text{S9F})$$

Finally, to estimate the error $\delta \rho_s$ in the measurement of density of the sample ρ_s one needs to substitute the expressions for the partial derivatives (eqns. S9A-F) into eqn. (S6) and evaluate the resulting expression using the values of parameters B_0 , d , χ_s , T , c , and h , and the tolerances of these parameters $\delta B_0 = \pm 0.003$ (T), $\delta d = \pm 0.5 \times 10^{-3}$ (m), $\delta \chi_s = \pm 10^{-5}$ (unitless), $\delta T = 1$ ($^{\circ}\text{K}$), $\delta c = \pm 0.002$ (M), and $\delta h = \pm 0.5 \times 10^{-3}$ (m), respectively. We provide two examples of successful application of this procedure below.

Dependence of $\delta \rho_s$ on δT .

We use eqn. (S6) (and eqns. (S9A-F)) to compare the effect of the uncertainty in temperature (± 1 $^{\circ}\text{C}$ vs. ± 10 $^{\circ}\text{C}$) on the accuracy of the measurement of density of polystyrene spheres levitating in 350 mM solution of MnCl_2 . This type of estimate may be appropriate for a user in a remote location with no immediate access to a thermometer for measuring temperature. For simplicity, we assume that all temperature-independent parameters and the value of B_0 are known *exactly*. **Table S1** summarizes the parameters used for and the result of this estimation.

Dependence of $\delta \rho_s$ on δB_0 .

Similarly, we use eqn. (S6) (and eqns. (S9A-F)) to compare the effect of $\delta B_0 = \pm 0.003$ T (typical accuracy established by using a magnetometer) vs. $\delta B_0 = 0.1$ T (a safe assumption for variation between NdFeB magnets supplied by various manufacturers) on the accuracy of the density measurement of polystyrene spheres levitating in 350 mM MnCl_2 . For simplicity, we

assume that all parameters except δB_0 are known exactly. **Table S2** summarizes the parameters used for and the result of this estimation – while the uncertainty in the measurement of density increases significantly from ± 0.0002 to 0.008 g/cm^3 , the measurement of density is still fairly accurate and may be useful in less demanding situations.

Table S0. Densities common plastics and organic liquids that can be used as density standards in magnetic levitation.

sample	density (g/cm ³)
organic polymers	
high-density polyethylene	0.97
polydimethylsiloxane (PDMS)	1.04
polystyrene	1.05
poly(styrene- <i>co</i> -acrylonitrile)	1.08
poly(styrene- <i>co</i> -methylmethacrylate)	1.14
nylon 6/6	1.14
polymethylmethacrylate	1.18
polycarbonate	1.22
neoprene rubber	1.23
polyethylene terephthalate (mylar)	1.40
polyvinylchloride (PVC)	1.40
cellulose acetate	1.42
polyoxymethylene (delrin)	1.43
polyvinyledene chloride	1.70
polytetrafluoroethylene (Teflon)	2.20
organic liquids	
3-fluorotoluene	0.997
2-fluorotoluene	1.004
fluorobenzene	1.025
3-chlorotoluene	1.072
chlorobenzene	1.107
2,4-difluorotoluene	1.120
2-nitrotoluene	1.163
nitrobenzene	1.196
1-chloro-2-fluorobenzene	1.244
1,3-dichlorobenzene	1.288
1,2-dichlorobenzene	1.305
dichloromethane	1.325
3-bromotoluene	1.410
bromobenzene	1.491
chloroform	1.492
1-bromo-4-fluorobenzene	1.593
hexafluorobenzene	1.612
carbon tetrachloride	1.630
tetradecafluorohexane	1.669
2,5-dibromotoluene	1.895
perfluoro(methyldecalin)	1.950
iodomethane	2.280
tribromomethane	2.890

Table S1. Dependence of uncertainty in measurement of density $\delta\rho_s$ on uncertainty in δT .

Parameter P	Description	Magnitude of P	δP	δP
experimental parameters				
B_0	strength of magnetic field at the surface of the magnet	0.375 T	known exactly	known exactly
d	distance between magnets	45 mm	known exactly	known exactly
T	Temperature	23 °C	± 1 °C	± 10 °C
c	Concentration of MnCl ₂	0.350 M	known exactly	known exactly
Unknowns				
χ_s	bulk magnetic susceptibility of the sample	-5×10^{-6} (SI, unitless)	known exactly	known exactly
calculated parameters				
$\rho_m(c, T)$	density of paramagnetic medium	1.0339 g/cm ³	± 0.0003 g/cm ³	± 0.0026 g/cm ³
$\chi_m(c, T)$	bulk magnetic susceptibility of the medium	$\pm 56 \times 10^{-6}$	known exactly	known exactly
constants				
g	acceleration due to gravity	9.80 m/s ²	n/a	n/a
μ_0	permeability of free space	$4\pi \times 10^{-6}$ N·A ⁻²	n/a	n/a
independent variable				
h	“levitation height” of the sample above the bottom magnet	12.0 mm	known exactly	known exactly
dependent variable				
ρ_s	density of sample	1.0483 g/cm ³	± 0.0003 g/cm ³	± 0.003 g/cm ³

Table S2. Dependence of uncertainty in measurement $\delta\rho_s$ on uncertainty in δB_0 .

Parameter P	Description	Magnitude of P	δP	δP
experimental parameters				
B_0	strength of magnetic field at the surface of the magnet	0.375 T	± 0.003 T	± 0.1 T
d	distance between magnets	45 mm	known exactly	known exactly
T	Temperature	23 °C	known exactly	known exactly
c	concentration of MnCl ₂	0.350 M	known exactly	known exactly
Unknowns				
χ_s	bulk magnetic susceptibility of the sample	-5×10^{-6} (SI, unitless)	known exactly	known exactly
calculated parameters				
$\rho_m(c, T)$	density of paramagnetic medium	1.0339 g/cm ³	known exactly	known exactly
$\chi_m(c, T)$	bulk magnetic susceptibility of the medium	$\pm 56 \times 10^{-6}$	known exactly	known exactly
constants				
g	acceleration due to gravity	9.80 m/s ²	n/a	n/a
μ_0	permeability of free space	$4\pi \times 10^{-6}$ N·A ⁻²	n/a	n/a
independent variable				
h	“levitation height” of the sample above the bottom magnet	12.0 mm	known exactly	known exactly
dependent variable				
ρ_s	density of sample	1.0483 g/cm ³	± 0.0002 g/cm ³	± 0.008 g/cm ³

Table S3. Linear correlations for h and ρ_s at different concentrations of MnCl_2 obtained from the plot of these two parameters in **Fig. 5A**.

concentration of $\text{MnCl}_2(\text{M})$	linear least squares fit to Fig. 5A	least squares linear fit in the form of Eqn 8	α	β
0.1	$h = -3664\rho_s + 3717$	$\rho_s = -0.0003h + 1.0145$	-0.0003	1.0145
0.5	$h = -449\rho_s + 493$	$\rho_s = -0.002h + 1.098$	-0.002	1.098
1.0	$h = -247\rho_s + 294$	$\rho_s = -0.004h + 1.190$	-0.004	1.190
1.5	$h = -160\rho_s + 206$	$\rho_s = -0.006h + 1.287$	-0.006	1.287
2.0	$h = -117\rho_s + 162$	$\rho_s = -0.008h + 1.385$	-0.008	1.385
2.5	$h = -93.4\rho_s + 138$	$\rho_s = -0.011h + 1.478$	-0.011	1.478
3.0	$h = -80.8\rho_s + 126$	$\rho_s = -0.012h + 1.559$	-0.012	1.559

Figure S6. Dependence of α and β on $[\text{MnCl}_2]$. A) Plot correlating α with $[\text{MnCl}_2]$; least squares linear fit: $y = 0.0041x - 0.0001$, $R^2 = 0.997$. B) Plot correlating β with $[\text{MnCl}_2]$; least squares linear fit: $y = 0.1889x + 1.0014$, $R^2 = 0.999$.

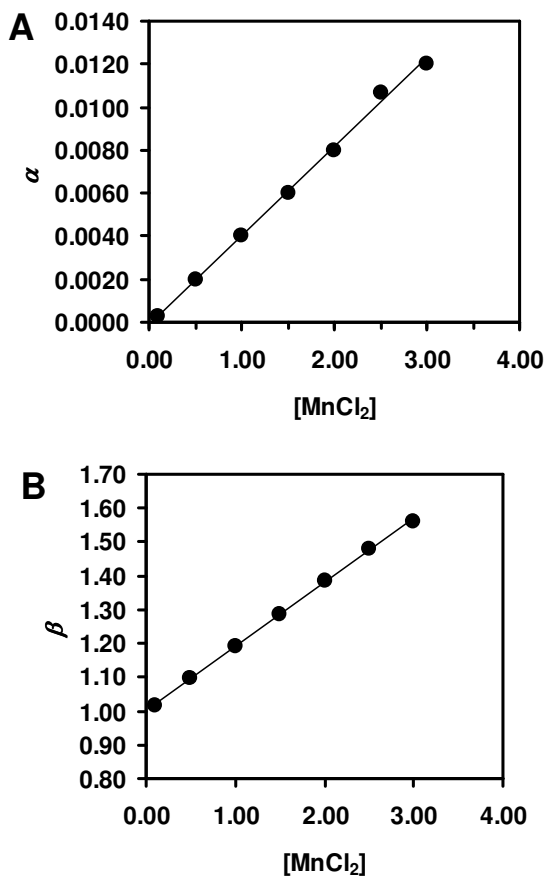


Table S4. Details of experimental parameters used for obtaining densities of solids using calibration curves in MnCl_2 . Table 2 compares the calculated densities from Tables S3-S5.

Sample	$[\text{MnCl}_2]$ (mol/L)	h (mm)	calibration curve used	Calculated ρ_s (g/cm^3)
glass beads				
$\rho = 1.0100 \text{ g/cm}^3$	0.100	16.7 ± 0.5	$h = -3664\rho_s + 3717$	1.0099 ± 0.0002
$\rho = 1.1000 \text{ g/cm}^3$	1.000	22.0 ± 0.5	$h = -247\rho_s + 294$	1.101 ± 0.002
$\rho = 1.1500 \text{ g/cm}^3$	1.500	21.9 ± 0.5	$h = -160\rho_s + 206$	1.152 ± 0.003
spherical polymers				
polystyrene	0.500	23.5 ± 0.5	$h = -449\rho_s + 493$	1.047 ± 0.001
nylon 6/6	1.000	13.5 ± 0.5	$h = -247\rho_s + 294$	1.137 ± 0.002
polymethylmethacrylate	2.000	23.3 ± 0.5	$h = -117\rho_s + 162$	1.186 ± 0.004
irregularly-shaped polymers				
polystyrene	0.500	22.0 ± 0.5	$h = -449\rho_s + 493$	1.049 ± 0.001
poly(styrene- <i>co</i> -acrylonitrile)	1.000	28.1 ± 0.5	$h = -247\rho_s + 294$	1.076 ± 0.002
poly(styrene- <i>co</i> -methylmethacrylate)	1.000	14.0 ± 0.5	$h = -247\rho_s + 294$	1.133 ± 0.002
organic droplets				
chlorobenzene	1.000	18.5 ± 0.5	$h = -247\rho_s + 294$	1.115 ± 0.003
2-nitrotoluene	1.000	5.2 ± 0.5	$h = -247\rho_s + 294$	1.169 ± 0.003
dichloromethane	3.000	19.5 ± 0.5	$h = -80.8\rho_s + 126$	1.324 ± 0.006
3-bromotoluene	3.000	12.5 ± 0.5	$h = -80.8\rho_s + 126$	1.405 ± 0.006
chloroform	3.000	6.5 ± 0.5	$h = -80.8\rho_s + 126$	1.479 ± 0.007

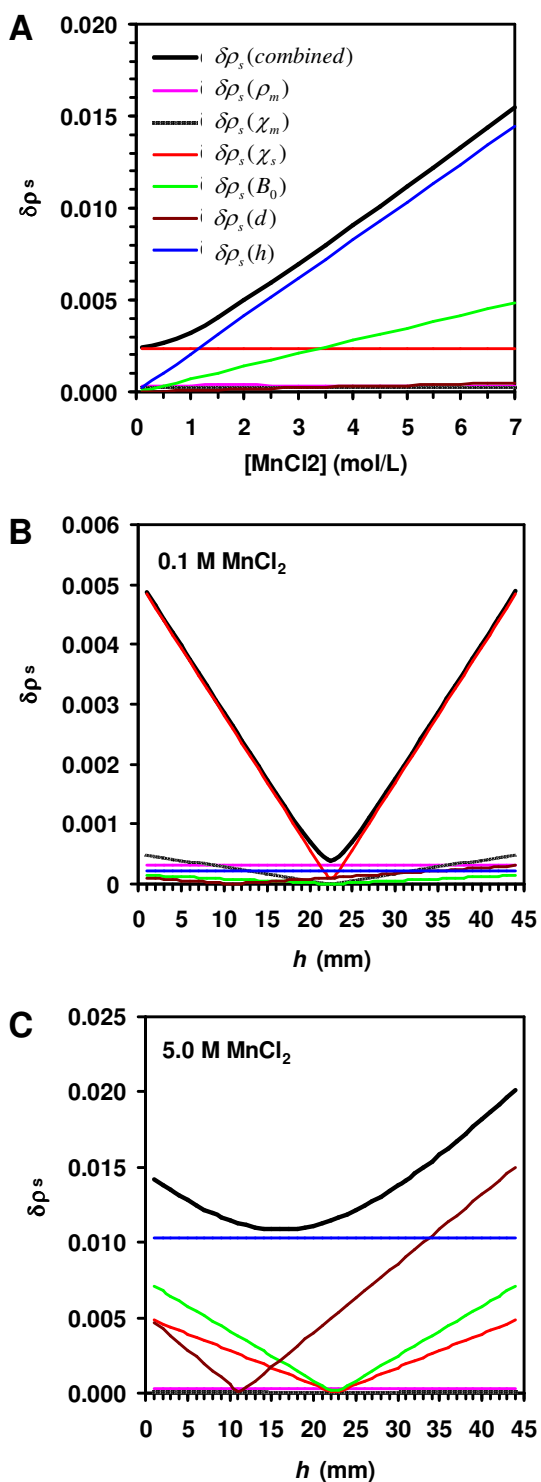
Table S5. Details of experimental parameters used for obtaining densities of solids using eqn (8) in MnCl_2 . Table 2 compares the calculated densities from Tables S3-S5.

Sample	$[\text{MnCl}_2]$ (mol/L)	Density of MnCl_2 solution (g/cm^3)	χ_m (unitless)	h (mm)	Calculated ρ_s (g/cm^3)
glass beads					
$\rho = 1.0100 \text{ g/cm}^3$	0.100	1.0081	10×10^{-6}	16.7 ± 0.5	1.010 ± 0.001
$\rho = 1.1000 \text{ g/cm}^3$	1.000	1.0994	177×10^{-6}	22.0 ± 0.5	1.101 ± 0.002
$\rho = 1.1500 \text{ g/cm}^3$	1.500	1.1486	270×10^{-6}	21.9 ± 0.5	1.152 ± 0.003
spherical polymers					
polystyrene	0.500	1.0492	84×10^{-6}	23.5 ± 0.5	1.047 ± 0.001
nylon 6/6	1.000	1.0994	177×10^{-6}	13.5 ± 0.5	1.134 ± 0.003
polymethylmethacrylate	2.000	1.1971	363×10^{-6}	23.3 ± 0.5	1.191 ± 0.005
irregularly-shaped polymers					
polystyrene	0.500	1.0492	84×10^{-6}	22.0 ± 0.5	1.050 ± 0.001
poly(styrene- <i>co</i> -acrylonitrile)	1.000	1.0994	177×10^{-6}	28.1 ± 0.5	1.078 ± 0.003
poly(styrene- <i>co</i> -methylmethacrylate)	1.000	1.0994	177×10^{-6}	14.0 ± 0.5	1.132 ± 0.003
organic droplets					
chlorobenzene	1.000	1.0994	177×10^{-6}	18.5 ± 0.5	1.115 ± 0.002
2-nitrotoluene	1.000	1.0994	177×10^{-6}	5.2 ± 0.5	1.166 ± 0.005
dichloromethane	3.000	1.2923	548×10^{-6}	19.5 ± 0.5	1.329 ± 0.007
3-bromotoluene	3.000	1.2923	548×10^{-6}	12.5 ± 0.5	1.416 ± 0.007
chloroform	3.000	1.2923	548×10^{-6}	6.5 ± 0.5	1.488 ± 0.008

Table S6. Details of experimental parameters used for obtaining densities of solids using eqn (8) in GdCl₃. Table 2 compares the calculated densities from Tables S3-S5.

Sample	[GdCl ₃] (mol/L)	Density of GdCl ₃ solution (g/cm ³)	χ_m (unitless)	h (mm)	Calculated ρ_s (g/cm ³)
glass beads					
$\rho = 1.0100 \text{ g/cm}^3$	0.050	1.0100	8×10^{-6}	26.0 ± 0.5	1.009 ± 0.001
$\rho = 1.1000 \text{ g/cm}^3$	0.400	1.0950	125×10^{-6}	20.2 ± 0.5	1.102 ± 0.002
$\rho = 1.1500 \text{ g/cm}^3$	0.600	1.1426	192×10^{-6}	20.5 ± 0.5	1.152 ± 0.003
spherical polymers					
polystyrene	0.200	1.0468	58×10^{-6}	22.8 ± 0.5	1.046 ± 0.001
nylon 6/6	0.400	1.0950	125×10^{-6}	8.6 ± 0.5	1.136 ± 0.004
polymethylmethacrylate	0.750	1.1780	242×10^{-6}	21.3 ± 0.5	1.185 ± 0.003
irregularly-shaped polymers					
polystyrene	0.140	1.0321	38×10^{-6}	8.5 ± 0.5	1.045 ± 0.003
poly(styrene- <i>co</i> - acrylonitrile)	0.250	1.0589	75×10^{-6}	12.7 ± 0.5	1.076 ± 0.003
poly(styrene- <i>co</i> - methylmethacrylate)	0.530	1.1260	168×10^{-6}	20.9 ± 0.5	1.132 ± 0.002
organic droplets					
chlorobenzene	0.500	1.1189	158×10^{-6}	27.7 ± 0.5	1.100 ± 0.003
2-nitrotoluene	0.500	1.1189	158×10^{-6}	12.2 ± 0.5	1.157 ± 0.003
dichloromethane	1.500	1.3514	493×10^{-6}	26.2 ± 0.5	1.300 ± 0.007
3-bromotoluene	1.500	1.3514	493×10^{-6}	17.8 ± 0.5	1.404 ± 0.006
chloroform	1.500	1.3514	493×10^{-6}	11.8 ± 0.5	1.471 ± 0.006

Figure S7. Plots showing the dependence of error in ρ_s on various experimental parameters at constant temperature. A) Dependence of $\delta\rho_s$ on $[\text{MnCl}_2]$ in water at 23 °C. Dependence of $\delta\rho_s$ on h over the entire vertical distance between the magnets (45mm) at B) 0.1 M MnCl_2 and C) 5.0 M MnCl_2 at 23 °C.



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- (13) Paramagnetic samples will also levitate in this device provided that $\chi_m > \chi_s$.